

Name

Class



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# Similarity and congruence

(9 – 1) Topic booklet

## Higher

These questions have been collated from previous years GCSE Mathematics papers.

**You must have:** Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out**.
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

### Information

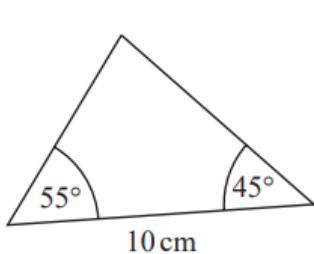
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

### Advice

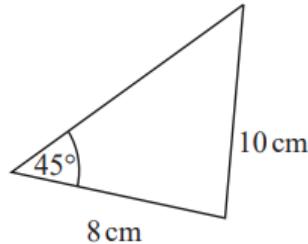
- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

**Answer ALL questions**  
**Write your answers in the space provided.**  
**You must write down all the stages in your working.**

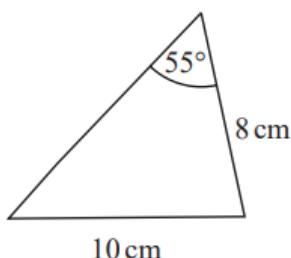
4 The diagram shows four triangles.



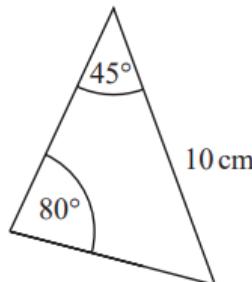
Triangle A



Triangle B



Triangle C



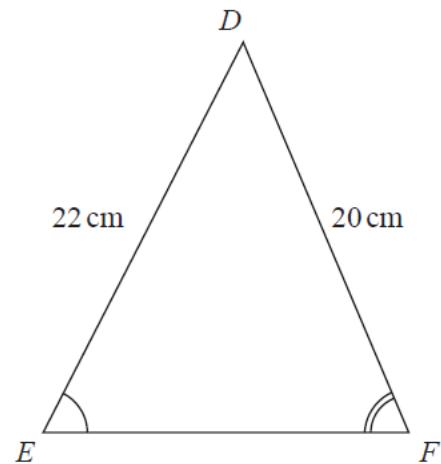
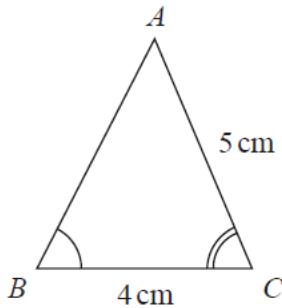
Triangle D

Two of these triangles are congruent.

Write down the letters of these two triangles.

..... and .....

5 Triangle  $ABC$  and triangle  $DEF$  are similar.



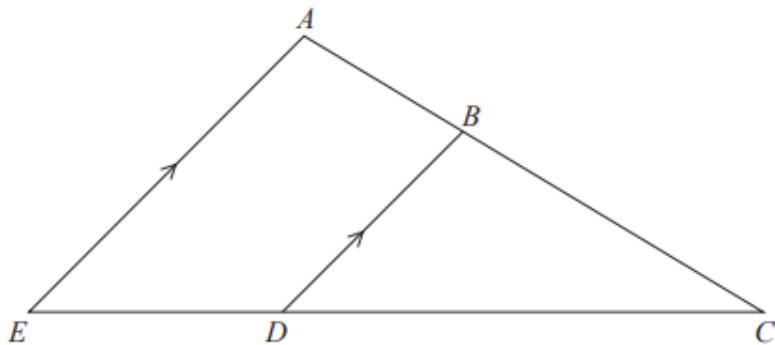
(a) Work out the length of  $EF$ .

..... cm  
(2)

(b) Work out the length of  $AB$ .

..... cm  
(2)

5



$ABC$  and  $EDC$  are straight lines.

$EA$  is parallel to  $DB$ .

$EC = 8.1$  cm.

$DC = 5.4$  cm.

$DB = 2.6$  cm.

(a) Work out the length of  $AE$ .

..... cm

(2)

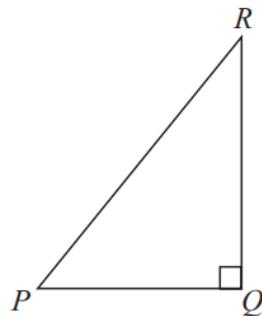
$AC = 6.15$  cm.

(b) Work out the length of  $AB$ .

..... cm

(2)

8 A playground is in the shape of a right-angled triangle.



Dan makes a scale drawing of the playground.

He uses a scale of 1 cm represents 5 m

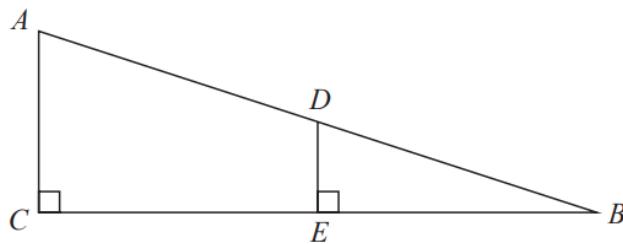
The area of the playground on the scale drawing is  $28 \text{ cm}^2$

The real length of  $QR$  is 40 m

Work out the real length of  $PQ$ .

..... m

10 The diagram shows two right-angled triangles  $ACB$  and  $DEB$ .



$$AD = 9 \text{ cm}$$

$$DE = 2 \text{ cm}$$

$$DB = 6 \text{ cm}$$

Calculate the length of  $CB$ .

Give your answer correct to 2 decimal places.

..... cm

**13** Solid **A** and solid **B** are similar.

The ratio of the height of solid **A** to the height of solid **B** is  $2:5$

The volume of solid **A** is  $12\text{ cm}^3$

Work out the volume of solid **B**.

.....  $\text{cm}^3$

13 Here are two similar solid shapes.



**A**



**B**



surface area of shape A : surface area of shape B = 3 : 4

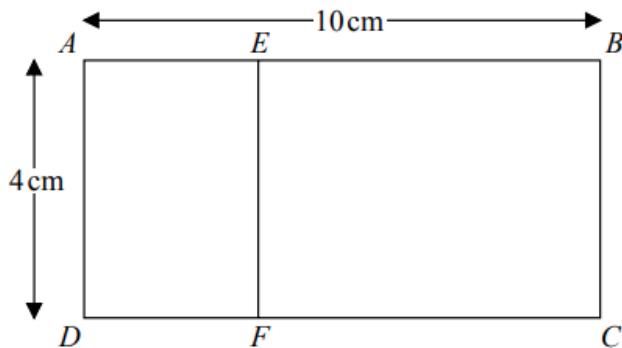
The volume of shape B is  $10\text{cm}^3$

Work out the volume of shape A.

Give your answer correct to 3 significant figures.

.....  $\text{cm}^3$

13 Rectangle  $ABCD$  is mathematically similar to rectangle  $DAEF$ .



$$AB = 10 \text{ cm.}$$

$$AD = 4 \text{ cm.}$$

Work out the area of rectangle  $DAEF$ .

.....  $\text{cm}^2$

14 Here are two squares, **A** and **B**.



**A**



**B**



The length of each side of square **B** is 4 cm greater than the length of each side of square **A**.  
The area of square **B** is 70 cm<sup>2</sup> greater than the area of square **A**.

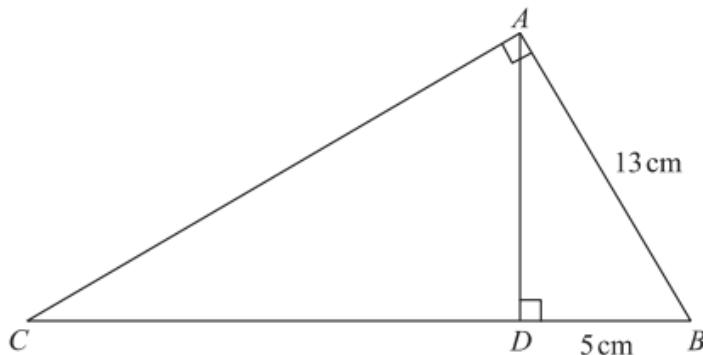
Find the area of square **B**.

Give your answer correct to 3 significant figures.

You must show all your working.

..... cm<sup>2</sup>

14  $ABC$  and  $ABD$  are two right-angled triangles.



Angle  $BAC$  = angle  $ADB$  =  $90^\circ$

$AB = 13$  cm

$DB = 5$  cm

Work out the length of  $CB$ .

..... cm

**14** Cone **A** and cone **B** are mathematically similar.

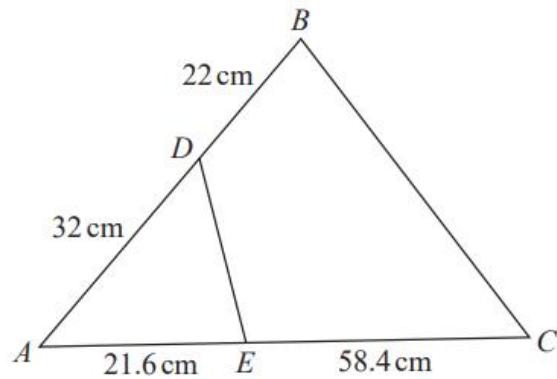
The ratio of the volume of cone **A** to the volume of cone **B** is  $27 : 8$

The surface area of cone **A** is  $297 \text{ cm}^2$

Show that the surface area of cone **B** is  $132 \text{ cm}^2$



15 The diagram shows triangle  $ABC$  and triangle  $AED$ .



Show that triangle  $ABC$  and triangle  $AED$  are similar.

**15** Three solid shapes **A**, **B** and **C** are similar.

The surface area of shape **A** is  $4 \text{ cm}^2$

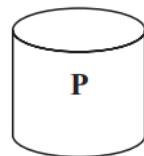
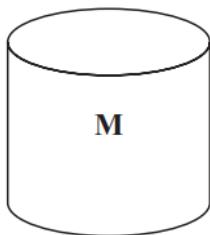
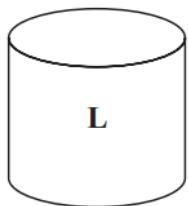
The surface area of shape **B** is  $25 \text{ cm}^2$

The ratio of the volume of shape **B** to the volume of shape **C** is  $27:64$

Work out the ratio of the height of shape **A** to the height of shape **C**.

Give your answer in its simplest form.

17 **L**, **M** and **P** are three similar solid cylinders made from the same material.



**L** has a mass of 64 g

**M** has a mass of 125 g

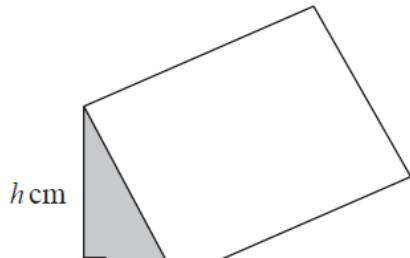
**M** has a total surface area of  $144 \text{ cm}^2$

**P** has a total surface area of  $16 \text{ cm}^2$

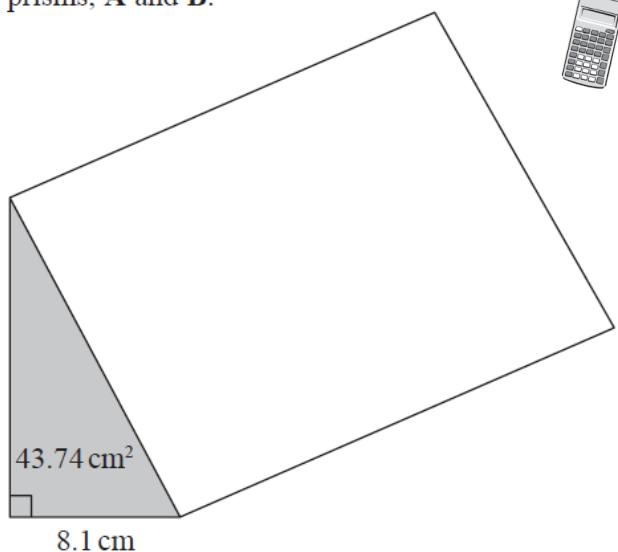
Work out

height of cylinder **L** : height of cylinder **M** : height of cylinder **P**

17 The diagram shows two similar solid triangular prisms, **A** and **B**.



Prism A



Prism B

The volume of prism A is  $58.806 \text{ cm}^3$

The volume of prism B is  $1587.762 \text{ cm}^3$

The cross section of each prism is a right-angled triangle.

For prism B

the length of the base of the triangle is 8.1 cm

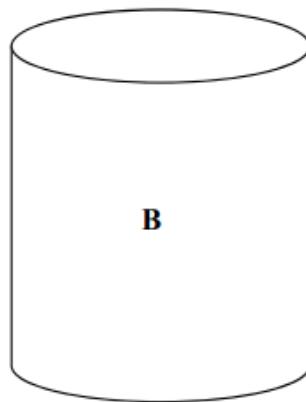
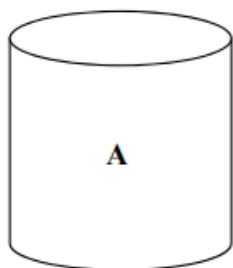
the area of the triangle is  $43.74 \text{ cm}^2$

The height of the triangle for prism A is  $h \text{ cm}$ .

Work out the value of  $h$ .

$$h = \dots$$

17 **A** and **B** are two similar cylindrical containers.



the surface area of container **A** : the surface area of container **B** = 4 : 9

Tyler fills container **A** with water.

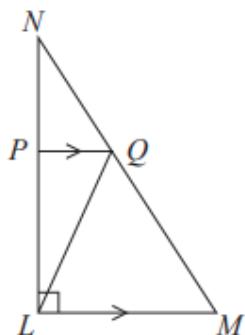
She then pours all the water into container **B**.

Tyler repeats this and stops when container **B** is full of water.

Work out the number of times that Tyler fills container **A** with water.

You must show all your working.

18 LMN is a right-angled triangle.



Angle  $NLM = 90^\circ$

$PQ$  is parallel to  $LM$ .

The area of triangle  $PNQ$  is  $8 \text{ cm}^2$

The area of triangle  $LPQ$  is  $16 \text{ cm}^2$

Work out the area of triangle  $LQM$ .

.....  $\text{cm}^2$

**18** Solid **A** and solid **B** are mathematically similar.

The ratio of the surface area of solid **A** to the surface area of solid **B** is 4:9

The volume of solid **B** is 405 cm<sup>3</sup>.

Show that the volume of solid **A** is 120 cm<sup>3</sup>.

19 **A, B and C** are three spheres.



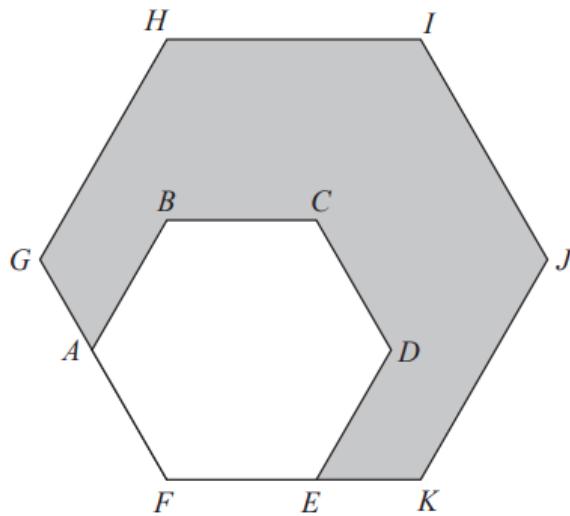
The volume of sphere **A** is  $125 \text{ cm}^3$

The volume of sphere **B** is  $27 \text{ cm}^3$

The ratio of the radius of sphere **B** to the radius of sphere **C** is  $1:2$

Work out the ratio of the surface area of sphere **A** to the surface area of sphere **C**.

19

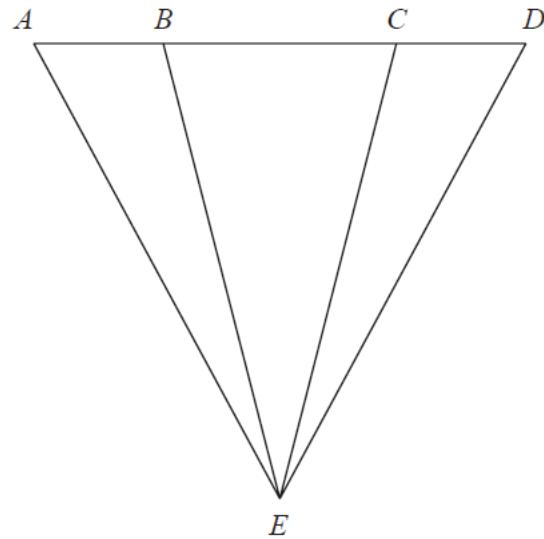


$ABCDEF$  is a regular hexagon with sides of length  $x$ .

This hexagon is enlarged, centre  $F$ , by scale factor  $p$  to give hexagon  $FGHIJK$ .

Show that the area of the shaded region in the diagram is given by  $\frac{3\sqrt{3}}{2}(p^2 - 1)x^2$

20 The diagram shows a triangle  $ADE$ .



$$AE = DE$$

$$AB : BC : CD = 1 : 2 : 1$$

Prove that triangle  $ACE$  is congruent to triangle  $DBE$ .

**20** Mark has made a clay model.

He will now make a clay statue that is mathematically similar to the clay model.



The model has a base area of  $6\text{cm}^2$

The statue will have a base area of  $253.5\text{cm}^2$

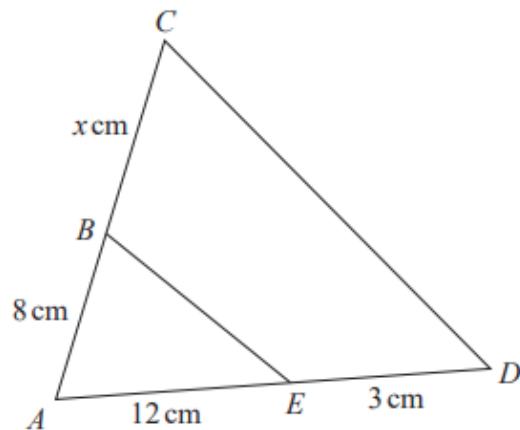
Mark used 2kg of clay to make the model.

Clay is sold in 10kg bags.

Mark has to buy all the clay he needs to make the statue.

How many bags of clay will Mark need to buy?

22 The two triangles in the diagram are similar.

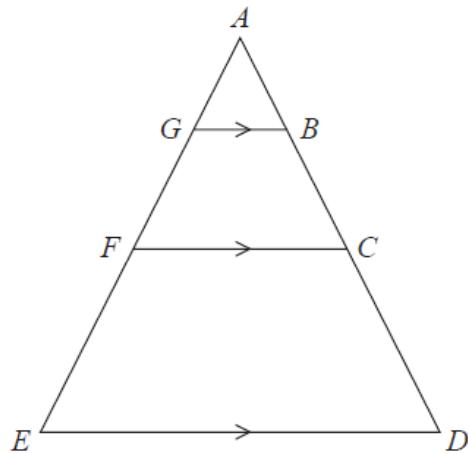


There are two possible values of  $x$ .

Work out each of these values.

State any assumptions you make in your working.

23 Here are three similar triangles,  $ABG$ ,  $ACF$  and  $ADE$ .



$ABCD$  and  $AGFE$  are straight lines.

$$AB:BC:CD = 1:2:3$$

Show that

$$\text{area of } ABG : \text{area of } BCFG : \text{area of } CDEF = 1 : 8 : 27$$